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BCS-012

BACHELOR OF COMPUTER APPLICATIONS (BCA) (Revised)

Term-End Examination

June, 2022

BCS-012 : BASIC MATHEMATICS

Time : 3 hours *Maximum Marks : 100*

Note : *Question number **1** is **compulsory**. Attempt any **three** questions from the remaining questions.*

1. (a) Solve the following system of linear equations using Cramer's rule : *5*

$$x + y = 0;\ \ y + z = 1;\ \ z + x = 3$$

(b) If 1, ω and ω^2 are cube roots of unity, show that

$$(2 - \omega)\,(2 - \omega^2)\,(2 - \omega^{10})\,(2 - \omega^{11}) = 49.$$ *5*

(c) Evaluate the integral $I = \int \frac{x^2}{(x+1)^3}\,dx$. *5*

(d) Solve the inequality $\frac{5}{|x - 3|} < 7$. *5*

(e) Show that $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (b - a)(c - a)(c - b)$. 5

(f) Find the quadratic equation whose roots are $(2 - \sqrt{3})$ and $(2 + \sqrt{3})$. 5

(g) Find the sum of an Infinite G.P., whose first term is 28 and fourth term is $\frac{4}{49}$. 5

(h) If z is a complex number such that $|z - 2i| = |z + 2i|$, show that $\text{Im}(z) = 0$. 5

2. (a) Evaluate $\lim_{x \to 0} \frac{\sqrt{1 + 2x} - \sqrt{1 - 2x}}{x}$. 5

(b) Prove that the three medians of a triangle meet at a point called centroid of the triangle which divides each of the medians in the ratio 2 : 1. 7

(c) A young child is flying a kite which is at a height of 50 m. The wind is carrying the kite horizontally away from the child at a speed of 6·5 m/s. How fast must the kite string be let out when the string is 130 m ? 8

3. (a) Using Principle of Mathematical Induction, show that $n(n + 1)(2n + 1)$ is a multiple of 6 for every natural number n. 5

(b) Find the points of local minima and local maxima for

$$f(x) = \frac{3}{4}x^4 - 8x^3 + \frac{45}{2}x^2 + 2015.$$ *5*

(c) Determine the 100^{th} term of the Harmonic Progression $\frac{1}{7}, \frac{1}{15}, \frac{1}{23}, \frac{1}{31}, ...$ *5*

(d) Find the length of the curve $y = 2x^{3/2}$ from the point (1, 2) to (4, 16). *5*

4. (a) Determine the shortest distance between

$$\vec{r}_1 = (1 + \lambda)\hat{i} + (2 - \lambda)\hat{j} + (1 + \lambda)\hat{k} \text{ and}$$

$$\vec{r}_2 = 2(1 + \mu)\hat{i} + (1 - \mu)\hat{j} + (-1 + 2\mu)\hat{k}.$$ *5*

(b) Find the area lying between two curves

$$y = 3 + 2x, \quad y = 3 - x, \quad 0 \leq x \leq 3,$$

using integration. *5*

(c) If $y = 1 + \ln(x + \sqrt{x^2 + 1})$, prove that

$$(x^2 + 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0.$$ *5*

(d) Find the angle between the lines

$$\vec{r}_1 = 2\hat{i} + 3\hat{j} - 4\hat{k} + t(\hat{i} - 2\hat{j} + 2\hat{k}) \text{ and}$$

$$\vec{r}_2 = 3\hat{i} - 5\hat{k} + s(3\hat{i} - 2\hat{j} + 6\hat{k}).$$ *5*

5. (a) If $A = \begin{bmatrix} -1 & 2 & 3 \\ 4 & 5 & 7 \\ 5 & 3 & 4 \end{bmatrix}$, show that A(adj A) = 0. *5*

(b) Use De-Moivre's theorem to find $(\sqrt{3} + i)^3$. *5*

(c) Show that $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$, for any two non-zero vectors $\vec{a}$ and $\vec{b}$. *5*

(d) If $y = ln\left[e^x \left(\frac{x-2}{x+2}\right)^{3/4}\right]$, find $\frac{dy}{dx}$. *5*
